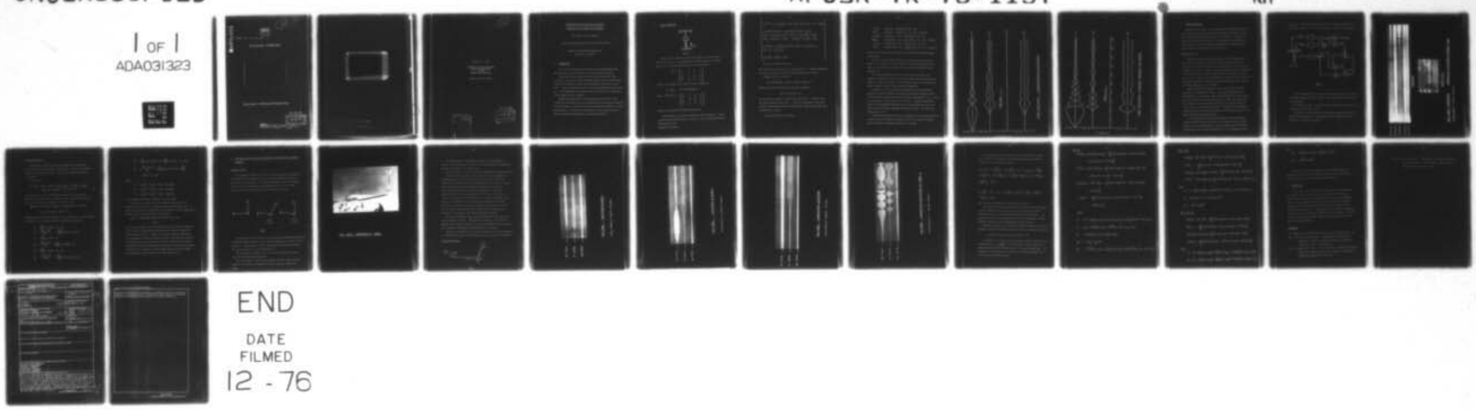
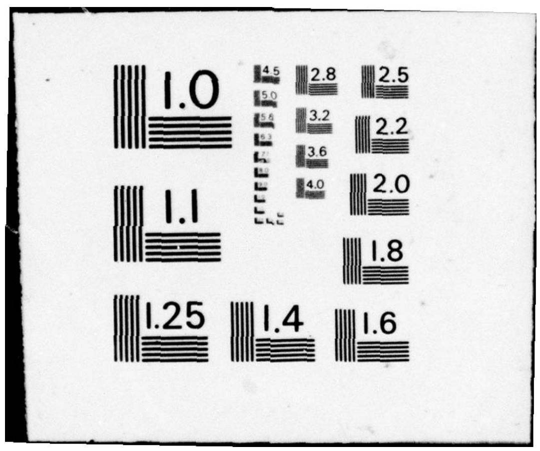
AD-A031 323 DUNDEE UNIV (SCOTLAND) DEPT OF MECHANICAL ENGINEERING F/G 20/11
PARAMETRIC AND NONLINEAR MODE INTERACTION BEHAVIOUR IN THE DYNA--ETC(U
1976 A D BARR, R P ASHWORTH AF-AFOSR-2723-74

UNCLASSIFIED AFOSR-TR-76-1131 NL

1 OF 1
ADA031323



END
DATE
FILMED
12 - 76


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0

(12) 82

AFOSR - TR - 76 - 1131

University of Dundee

Department of Mechanical Engineering

12

AFOSR - 74 - 2723A

PARAMETRIC AND NONLINEAR MODE INTERACTION BEHAVIOUR IN THE DYNAMICS OF STRUCTURES

Interim Scientific Report

PARAMETRIC AND NONLINEAR MODE INTERACTION BEHAVIOUR IN THE DYNAMICS OF STRUCTURES

A.D.S. Barr and R.P. Ashworth

Interim Scientific Report under Grant AFOSR-74-2723A

Department of Mechanical Engineering

University of Dundee

1. INTRODUCTION

This report describes the continuation of the investigation into the behaviour of structures under internal resonance conditions.

Completion of the analysis on the three mode interaction case, outlined in the previous report (September 1975), yields the variational equations describing the slowly varying amplitudes and phases of each mode. Computer simulation (C.S.M.P.) of these equations has been carried out. Both parametric and nonlinear terms are found to be of importance in the response of the structure.

Further experimental work on the original four mode model, where 'parametric' excitation of two of the modes leads to eventual participation of all the modes, is presented along with a theoretical approach of a more general nature. Attention is then turned to a less complex two-mode system exhibiting both parametric and nonlinear behaviour.

2. 3 MODE INTERACTION

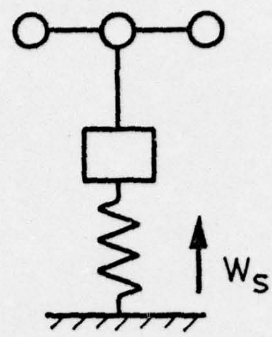


Fig. 1

Taking the simplified three mode model (Fig. 1), as described in the previous report, and transforming from the generalised coordinates W_1, V_2, S, A to normal coordinates $\underline{p}$ via the modal matrix

$$[X] = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \alpha_1 & 0 & \alpha_3 & 0 \\ 0 & \alpha_2 & 0 & \alpha_4 \end{bmatrix}$$

results in the equations of motion in normal coordinates,

$$\ddot{p} + [\Omega]^2 p = [X^{-1}]\ [M^{-1}] f(p, \dot{p}, \ddot{p}, t)$$

where $$[X^{-1}]\ [M^{-1}] = \begin{bmatrix} X_1 & 0 & X_2 & 0 \\ 0 & X_3 & 0 & X_4 \\ X_5 & 0 & X_6 & 0 \\ 0 & X_7 & 0 & X_8 \end{bmatrix}$$

and α_r, X_r are evaluated from system constants.

Upon setting $w_s = w_o \cos\Omega t$, introducing a small parameter $\varepsilon = w_o/\ell_2$, nondimensionalizing with respect to w_o, $\underline{N} = p/w_o$ and retaining quadratic nonlinearities only,

$f(\underline{N}, \dot{\underline{N}}, \ddot{N}, t)$ *becomes:-*

$$\begin{bmatrix} C_{11}\Omega^2\cos\Omega t - C_{12}\varepsilon\ell_2(N_2\ddot{N}_2 + N_2\ddot{N}_4 + N_4\ddot{N}_2 + N_4\ddot{N}_4 + \dot{N}_2{}^2 + \dot{N}_4{}^2 + 2\dot{N}_2\dot{N}_4) \\ \\ C_{21}\varepsilon\ell_2\Omega^2\cos\Omega t(N_2+N_4) - C_{21}\varepsilon\ell_2(\ddot{N}_1+\ddot{N}_3)(N_2+N_4) + C_{22}\varepsilon\ell_2 \\ ((\alpha_1\dot{N}_1 + \alpha_3\dot{N}_3)(\alpha_2\dot{N}_2 + \alpha_4\dot{N}_4) + \tfrac{1}{2}((\alpha_1N_1 + \alpha_3N_3)(\alpha_2\ddot{N}_2 + \alpha_4\ddot{N}_4) + \\ (\alpha_2N_2 + \alpha_4N_4)(\alpha_1\ddot{N}_1 + \alpha_3\ddot{N}_3))) - C_{23}\varepsilon\ell_2(N_2 + N_4)(\alpha_1\ddot{N}_1 + \alpha_3\ddot{N}_3) \\ \\ C_{31}\Omega^2\cos\Omega t + C_{32}\varepsilon\ell_2(\ddot{N}_2 + \ddot{N}_4)(\alpha_2N_2 + \alpha_4N_4) - C_{33}\varepsilon\ell_2((N_2 + N_4) \\ (\ddot{N}_2 + \ddot{N}_4) + (\dot{N}_2 + \dot{N}_4)^2) \\ \\ C_{41}\varepsilon\ell_2(\ddot{N}_2 + \ddot{N}_4)(\alpha_1N_1 + \alpha_3N_3) \end{bmatrix}$$

Where C_{ij} are system constants.

The equations of motion are now in normal mode form. Struble's Asymptotic method [1] is applied in order to obtain an approximate solution.

Taking solutions of the form

$$N_i = A_i(t)\cos(\omega_i t + \phi_i(t)) + \varepsilon a_i(t) + \varepsilon^2 b_i(t) + \ldots$$

imposing the internal and external resonance conditions

$$\omega_1 + \omega_2 = \omega_4 \text{ and } \Omega = \omega_4$$

and limiting the analysis to one of the first order in ε results in the so-called 'variational equations'. These are simultaneous first order nonlinear differential equations describing the slowly varying amplitudes and phases (A_i and ϕ_i).

In this case they take the form:-

$$
\begin{aligned}
-2A_1\omega_1\dot{\phi}_1 &= C_1\Omega^2\cos\phi_1 - C_2A_2A_4\cos(\phi_2 + \phi_4 - \phi_1) \\
-2\dot{A}_1\omega_1 &= C_3\Omega^2\sin\phi_1 - C_4A_2A_4\sin(\phi_2 + \phi_4 - \phi_1) + 2\varepsilon\zeta_1\omega_1{}^2A_1 \\
-2A_2\omega_2\dot{\phi}_2 &= C_5\Omega^2A_4\cos(\phi_4 + \phi_2) - C_6A_1A_4\cos(\phi_2 + \phi_4 - \phi_1) \\
-2\dot{A}_2\omega_2 &= C_7\Omega^2A_4\sin(\phi_4 + \phi_2) - C_8A_1A_4\sin(\phi_2 + \phi_4 - \phi_1) + 2\varepsilon\zeta_2\omega_2{}^2A_2 \\
-2A_4\omega_4\dot{\phi}_4 &= C_9\Omega^2A_2\cos(\phi_4 + \phi_2) - C_{10}A_1A_2\cos(\phi_2 + \phi_4 - \phi_1) \\
-2\dot{A}_4\omega_4 &= C_{11}\Omega^2A_2\sin(\phi_4 + \phi_2) - C_{12}A_1A_2\sin(\phi_2 + \phi_4 - \phi_1) + 2\varepsilon\zeta_4\omega_4{}^2A_4
\end{aligned}
$$

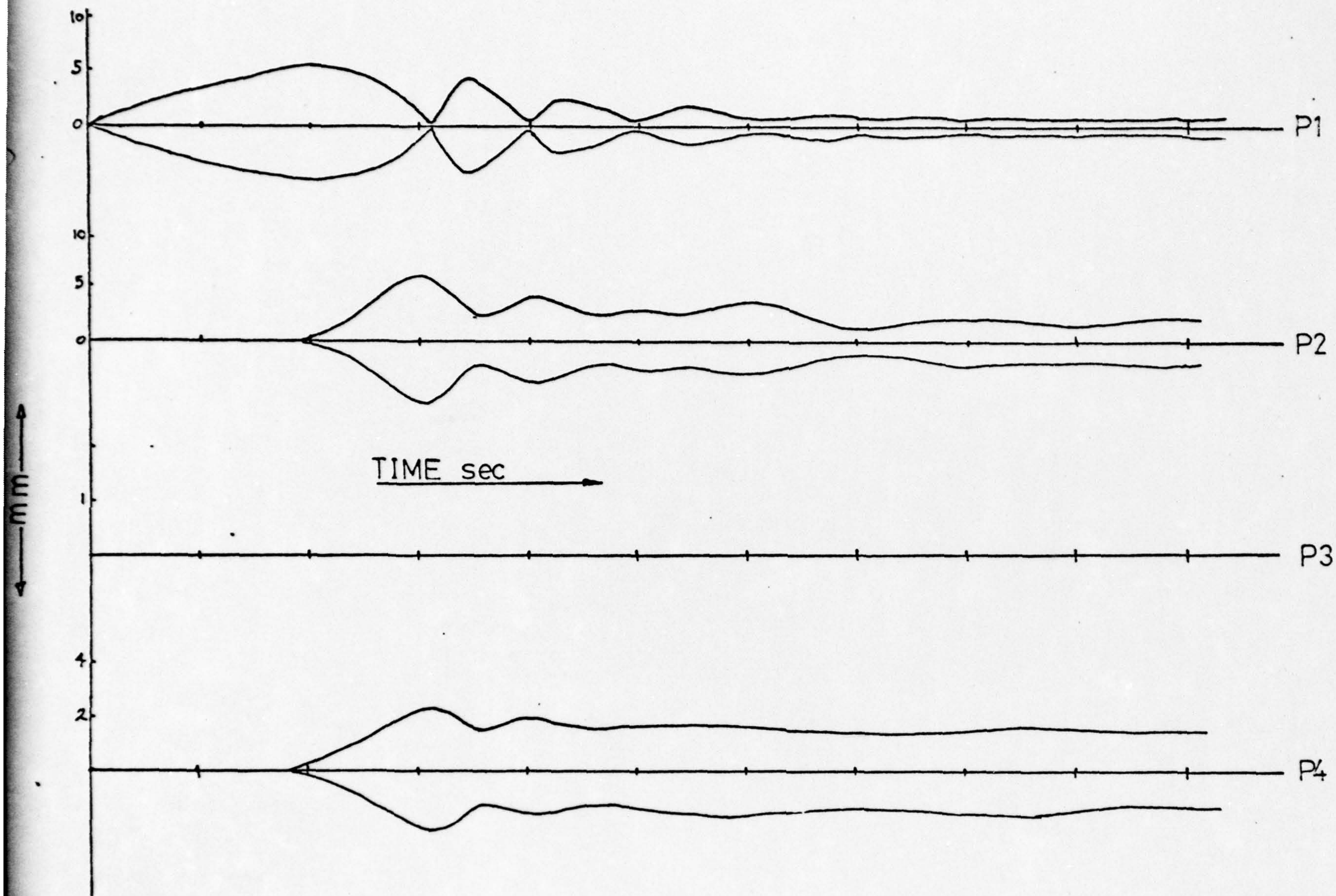
where C_n are constants for a given ε, and ζ_n are introduced modal damping coefficients.

The terms involving phase angle ϕ_1 only arise from direct forcing terms in the equations of motion, whilst the terms involving phase angles $(\phi_2 + \phi_4 - \phi_1)$ and $(\phi_4 + \phi_2)$ arise from nonlinear and parametric terms respectively.

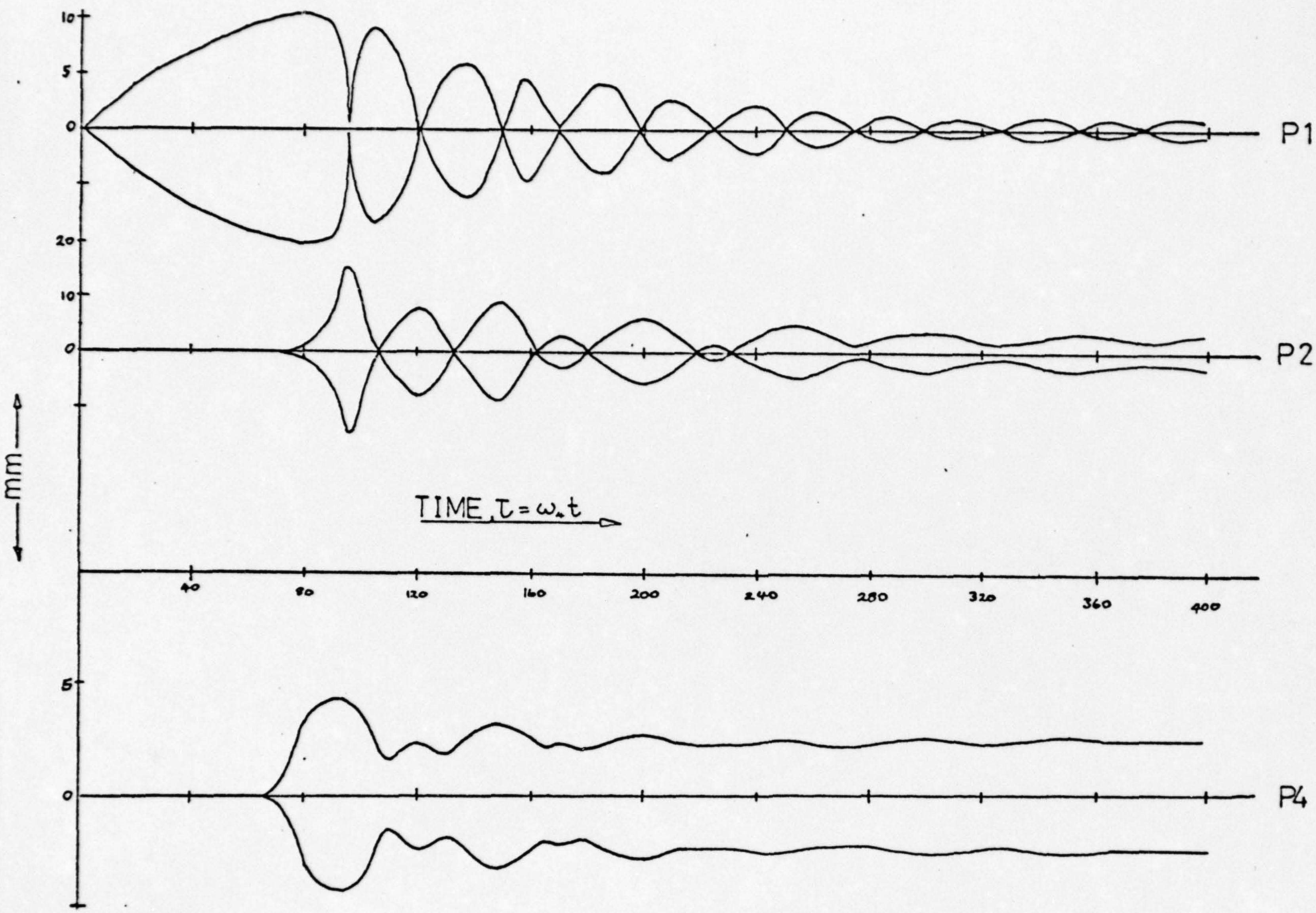
The standard procedure now is to set the left hand sides of the variational equations to zero to give steady state response amplitudes [2]. However an analytical solution to the equations appears to be unobtainable and so recourse is made to their numerical integration.

On pages 5, 6 are the results of integration of both the complete equations of motion and their corresponding variational equations for the model with exact internal and external frequency relationships. (ω_4 = 6.20 Hz, ω_3 = 6.93 Hz, ω_2 = 22.26 Hz, $\omega_1 = \Omega$ = 28.46 Hz). Damping coefficients, obtained from logarithmic decrement tests on the experimental model, have been introduced through a Rayleigh Dissipation Function. The excitation amplitude is 0.1 mm.

Although the transient behaviour is different, the variational equations do show the mode interaction taking place and the convergence to steady state conditions of comparable amplitude in each mode.



CSMP SIMULATION quadratic equations with damping



CSMP SIMULATION variational equations with damping

3. 4 MODE INTERACTION

The equations of motion for the four degree of freedom model (page 15 of previous interim report) contain both nonlinear and parametric terms, and in general these will always coexist in a structure. Indeed, the conditions which make the nonlinear terms important in the behaviour of the structure are also the conditions under which the parametric terms become important, as shown in the previous section.

Experimental Work

Further experimental work on the four mode model has shown that it is possible to excite the two symmetric modes, fuselage bending (P_4) symmetric tail bending (P_2) by forcing 'parametrically' at the combination frequency $\Omega = \omega_2 + \omega_4$. Growth of the fourth mode then leads to interaction with the first and third modes through the internal resonance condition $\omega_4 = \omega_3 + \omega_1$. Hence we have an interesting situation in which the structure is excited harmonically far away from any natural frequency but responds with large amplitudes in all its modes.

Signal conditioning equipment has been custom built to show the behaviour of the model more clearly. The model is monitored by an accelerometer on the fuselage under the fin along with the strain gauge pairs at the roots of each beam forming the 'T' tail. The signals from these represent the generalised coordinates of the mathematical model used to generate the equations of motion. Symmetric and antisymmetric tailplane motions are monitored by summing and differencing the two tail-plane signals using analogue devices.

It was found that the accelerometer signal and the sum tailplane signal were fairly representative of the two symmetric modes P_2 and P_4. However, the fin and antisymmetric tailplane signals required further manipulation to give signals representative of the antisymmetric modes

P_1 and P_3. Effectively the analogue devices do the same transformation as the modal matrix. Fig. 2 shows the schematic layout of the equipment.

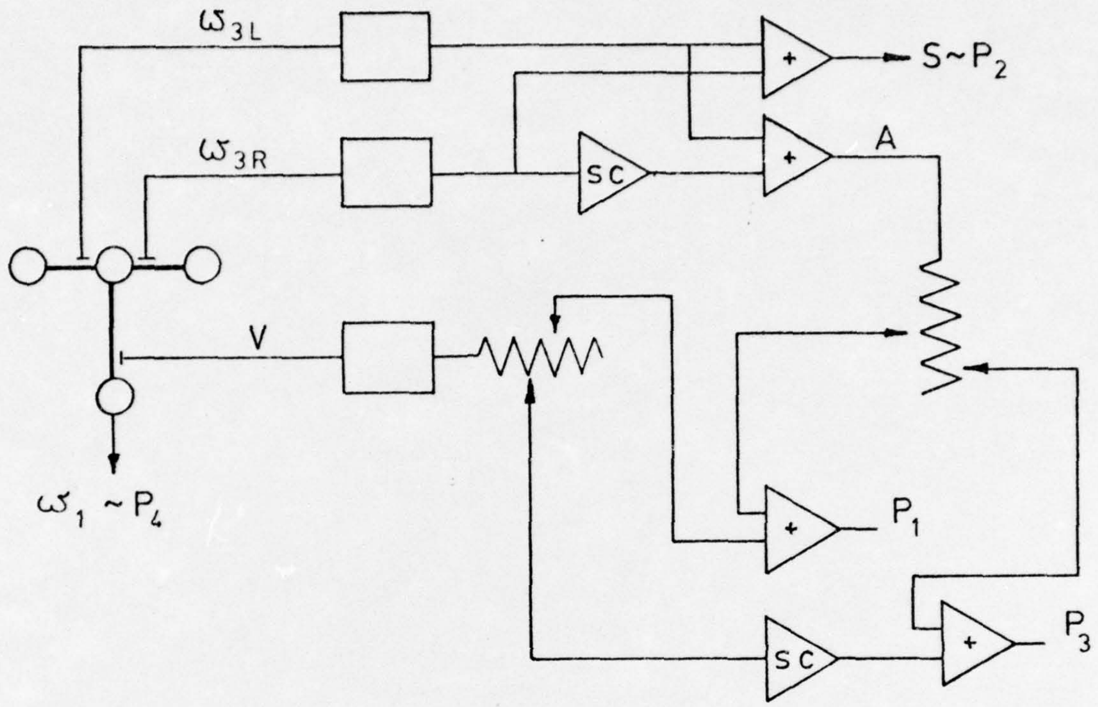
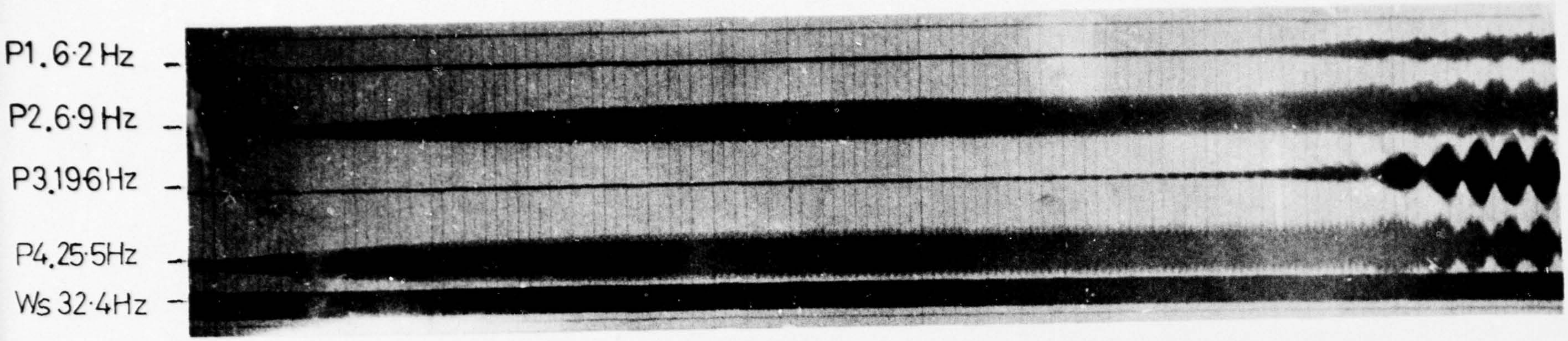


Fig. 2

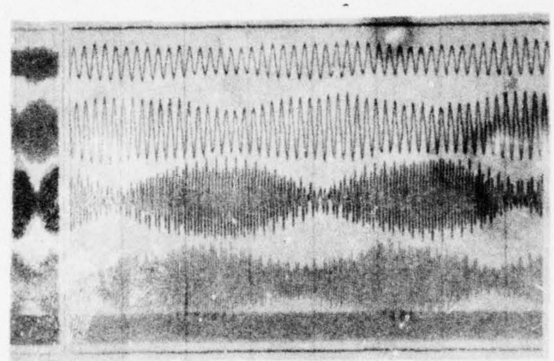
The variable resistors are adjusted until the signals P_1 and P_3 contain the single frequencies ω_1 and ω_3 only. These are then output to an ultra-violet recorder.

The results on page 9 show the U-V record obtained when the excitation frequency is 32.4 Hz (6.9 + 25.5).

The growth of the symmetric modes and eventual interaction in all the modes can be clearly seen. Quasi-steady state conditions with a beat period of 4.2 seconds are observed.



TRANSIENT



STEADY STATE

FOUR MODE INTERACTION

COMBINATION EXCITATION OF MODES 2 & 4

timing marks at 1 second intervals

Theoretical Approach

The simplified mathematical model developed for the three mode interaction problem is inadequate in this case as it lacks the parametric terms on the two symmetric modes. Hence a more general approach has been adopted.

The equation

$$\ddot{p}_r + \Omega_r^2 p_r = (\Omega_r^2 - \omega_r^2)p_r + \varepsilon\{k_{rj}p_j\cos\Omega_f t + l_{rij}p_i\ddot{p}_j + m_{rij}\dot{p}_i\dot{p}_j + \Omega_r d_{rr}\dot{p}_r + F_r\cos\Omega_f t\} \quad \text{(no sum on r)} \tag{1}$$

is the general form of the r^{th} normal mode equation with parametric, quadratic nonlinear inertial, damping and direct forcing terms included. $\Omega_r \simeq \omega_r$ is the response frequency of the r^{th} mode [3].

Application of Struble's method, assuming solutions of the form

$$p_r = A_r(t)\cos(\Omega_r t + \phi_r(t)) + \varepsilon X_1$$

imposing the resonance conditions $\Omega_f = \Omega_2 + \Omega_4$ and $\Omega_1 + \Omega_3 = \Omega_4$ and limiting the analysis to one of first order in ε results in the equations

$$\dot{A}_1 = \varepsilon \frac{A_3A_4}{4\Omega_1} L_1 \sin(\phi_4 - \phi_3 - \phi_1)$$

$$\dot{\phi}_1 = -\frac{(\Omega_1^2 - \omega_1^2)}{2\Omega_1} - \varepsilon \frac{A_3A_4}{4A_1\Omega_1}\cos(\phi_4 - \phi_3 - \phi_1)$$

$$\dot{A}_2 = \varepsilon \frac{A_4}{4\Omega_2} k_{24} \sin(\phi_2 + \phi_4)$$

$$\dot{\phi}_2 = -\frac{(\Omega_2^2 - \omega_2^2)}{2\Omega_2} + \varepsilon \frac{A_4}{4A_2\Omega_2} k_{24} \cos(\phi_2 + \phi_4)$$

$$\dot{A}_3 = \varepsilon \frac{A_1A_4}{4\Omega_3} L_3 \sin(\phi_4 - \phi_3 - \phi_1)$$

$$\dot{\phi}_3 = -\frac{(\Omega_3^2 - \omega_3^2)}{2\Omega_3} - \varepsilon \frac{A_1A_4}{4A_3\Omega_3} L_3 \cos(\phi_4 - \phi_3 - \phi_1)$$

$$\dot{A}_4 = \varepsilon\{\frac{A_2}{4\Omega_4} k_{42} \sin(\phi_2 + \phi_4) - \frac{A_1 A_3}{4\Omega_4} L_4 \sin(\phi_4 - \phi_3 - \phi_1)\}$$

$$\dot{\phi}_4 = -\frac{(\Omega_4{}^2 - \omega_4{}^2)}{2\Omega_4} + \varepsilon\{ \frac{A_2}{4A_4\Omega_4} k_{42} \cos(\phi_2 + \phi_4) - \frac{A_1 A_3}{4A\Omega_4}$$

$$\cos(\phi_4 - \phi_3 - \phi_1)\}$$

where

$$L_1 = \ell_{143}\Omega_3{}^2 + \ell_{134}\Omega_4{}^2 - (M_{143} + M_{134})\Omega_3\Omega_4$$

$$L_3 = \ell_{341}\Omega_1{}^2 + \ell_{314}\Omega_4{}^2 - (M_{341} + M_{314})\Omega_1\Omega_4$$

$$L_4 = \ell_{413}\Omega_3{}^2 + \ell_{431}\Omega_1{}^2 - (M_{413} + M_{431})\Omega_1\Omega_3$$

and damping terms have been excluded for the time being.

As can be seen, modes 1 and 3 have nonlinear terms, mode 2 has parametric terms and mode 4 has both parametric and nonlinear terms. No steady state solution is expected from these equations, but the experimental evidence points to 'quasi-steady state' solution.

Hence as a first approximation a solution of the form

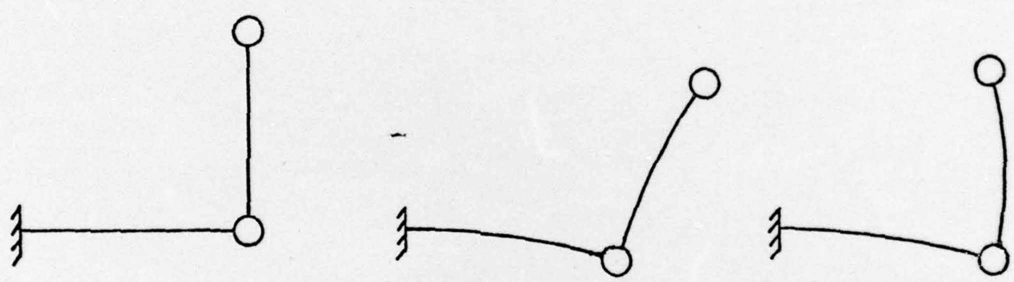
$$\dot{\phi}_r = 0$$

$A_r = C_{ro} + C_{r1} \cos(\beta t + \gamma_r)$ was substituted into the variational equations in the hope of finding analytical expressions for the beat frequency β and the amplitude ratios. However only trivial solutions resulted, and as no mathematical model was readily available to give physical parameters for a simulation of the variational equations, attention was focussed on a simpler two mode model exhibiting both parametric and nonlinear behaviour. It was felt that the solution of this simpler system would give insight into appropriate solution forms for the more complex case.

4. TWO MODE SYSTEM UNDER DIRECT AND PARAMETRIC EXCITATION WITH INTERNAL RESONANCE

Experimental Work

The photograph on page 13 is of the two mode model under investigation. It differs from the original two mode model in that bending deflections occur all in the same plane. By careful adjustment of masses and lengths the two natural frequencies have been arranged to be in a 2 - 1 ratio. Fig. 3 shows the natural frequencies and mode shapes obtained.



Mode 1
8.75 Hz

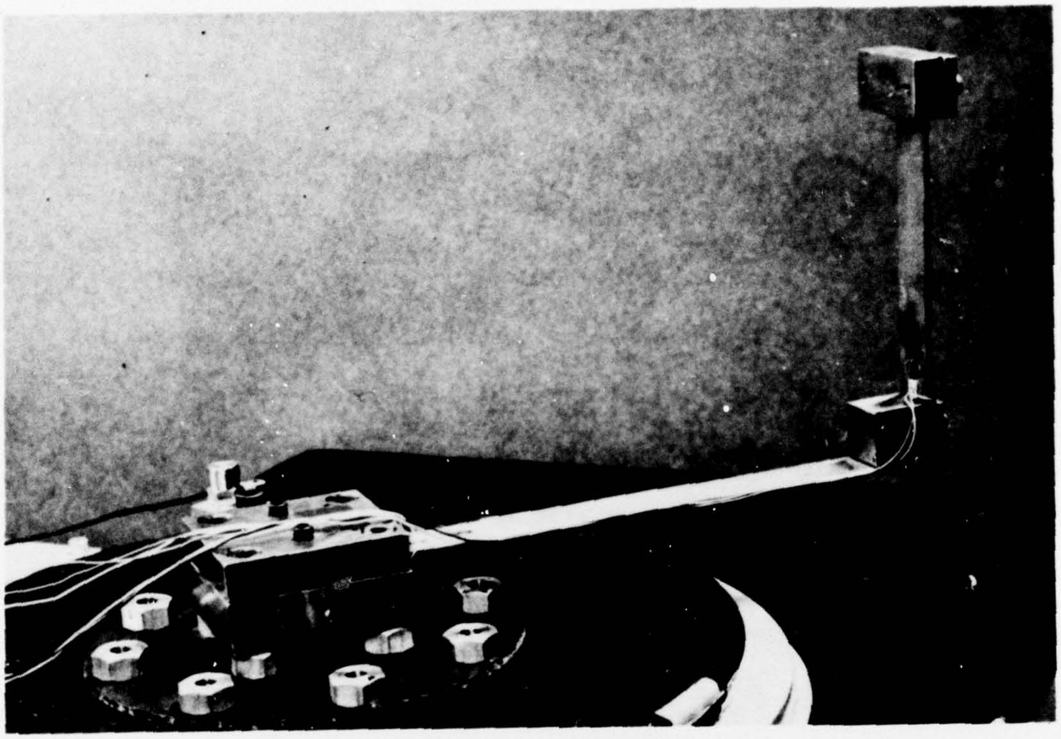
Mode 2
17.5 Hz

Fig. 3

Strain gauge pairs at the roots of each beam provide signals representative of the generalised coordinates to be used in the theoretical analysis of the structure. Transformation to normal coordinates is achieved by using the same analogue devices as in the four mode case.

The photographs on pages 15, 16, 17, 18 are of ultraviolet records taken from the experimental apparatus.

The first, page15 , shows direct forcing of the first mode at 8.75 Hz. Growth of the second mode, through the internal resonance condition is rapid.



TWO MODE EXPERIMENTAL MODEL

The second, page 16, shows direct forcing on the second mode at 17.5 Hz. This again leads to growth in the first mode and a corresponding decrease in amplitude in the second mode.

The third, page 17, shows forcing at the combination frequency, 26.4 Hz.

Excitation level is high in order to achieve large amplitudes for nonlinear effects to become dominant and the record starts with the ordinary forced response, at the forcing frequency, of each mode. There is then a transformation in the response as each mode is also excited parametrically and nonlinear interaction takes place. Work done previously by Asmis & Tso [4], suggests that for such a system there is a continuous interchange of energy between the participating modes. This however is not apparent here and steady state conditions are achieved. It should be noted that the forcing frequency not only equals the sum of the two natural frequencies but also is three times the first natural frequency, a consequence of the internal resonance condition.

The fourth photograph, page 18, shows excitation at twice the second natural frequency, 35.2 Hz. This leads to excitation of the high mode parametrically, leading to growth in the low mode through internal resonance and a continuous interchange of energy between the two modes.

Again, as a consequence of the internal resonance, the forcing frequency is four times the lower natural frequency as well as being twice the higher.

Theoretical Analysis

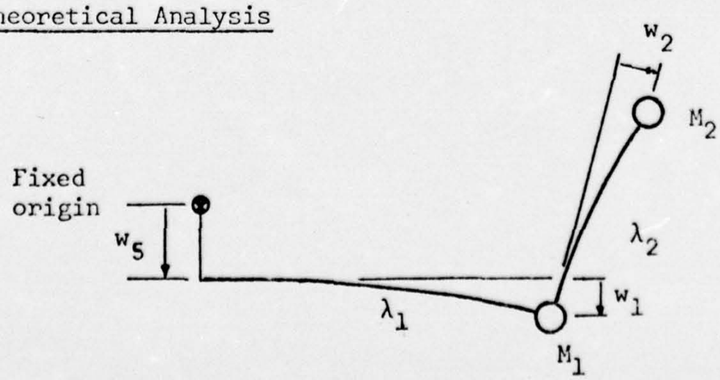
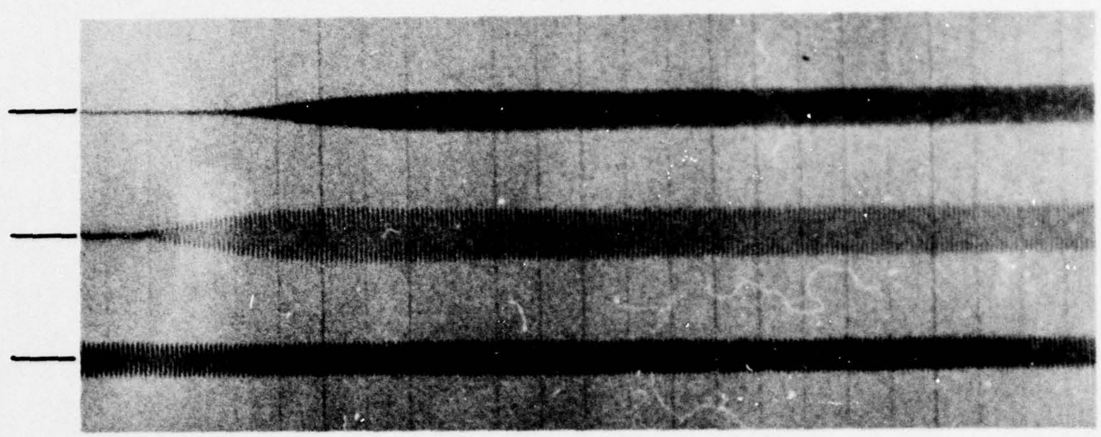
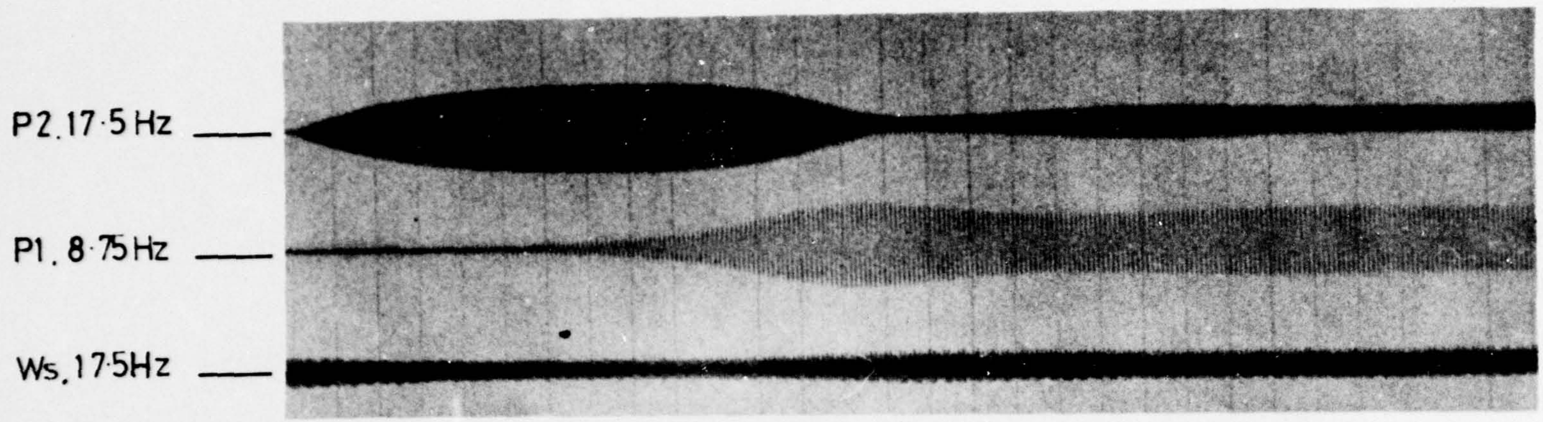


Fig. 4



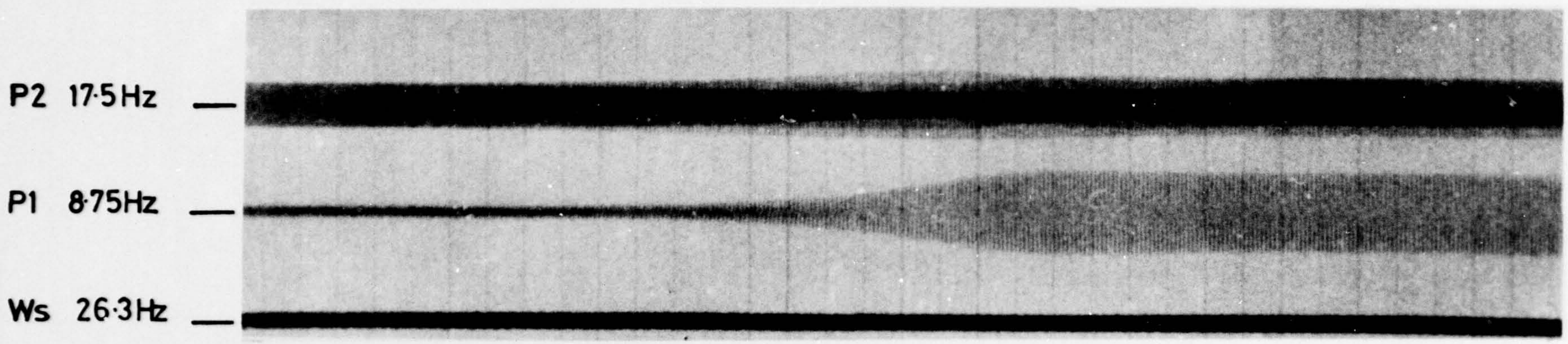
TWO MODE EXCITATION OF MODE 1

timing marks at 1 second intervals



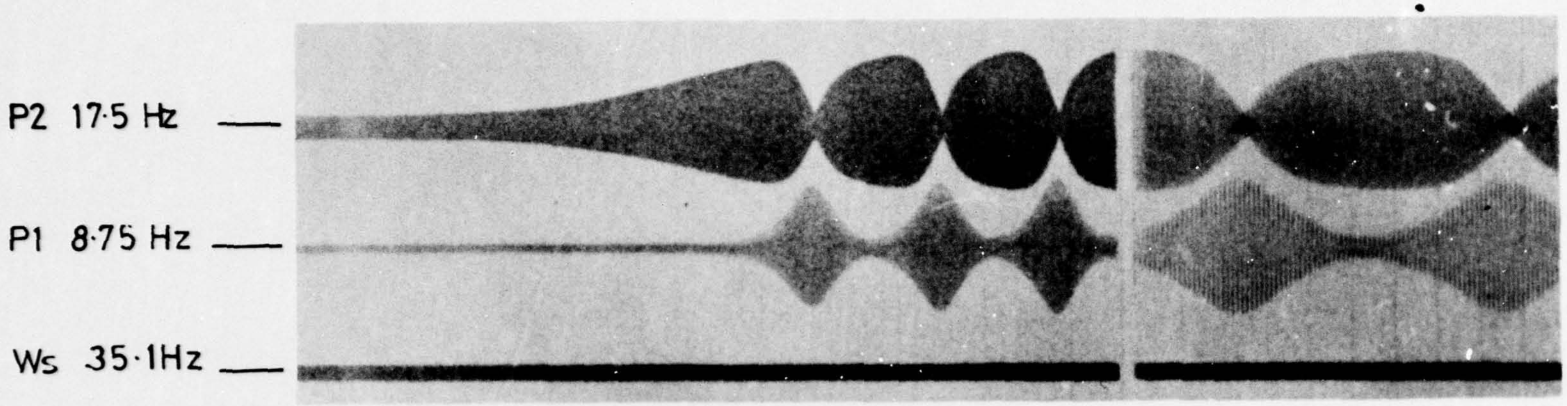
TWO MODE EXCITATION OF MODE 2

timing marks at 1 second intervals



<u>TWO MODE COMBINATION EXCITATION</u>

timing marks at 1 second intervals



TWO MODE PARAMETRIC EXCITATION OF MODE 2

timing marks at 1 second intervals

By approximating the structure with a two mode lumped mass model, with coordinates as in Figure 4, and applying Lagrange's procedure to it results in the equations of motion:-

$$\{M_1 + M_2(1 + 2.25\frac{\ell^2_2}{\ell^2_1})\}\ddot{w}_1 + (M_2 1.5\frac{\ell_2}{\ell_1})\ddot{w}_2 + \lambda_1 w_1 = -\{M_1+M_2(1 + 2.25\frac{\ell_2}{\ell^2_1}w_1$$

$$+ \frac{1.5}{\ell_1}w_2)\}\ddot{w}_s - M_2\{0.9\frac{\ell_2}{\ell^2_1}w_1\ddot{w}_1 + 0.45\frac{\ell_2}{\ell^2_1}\dot{w}_1{}^2 + \frac{1.2}{\ell_2}(w_2\ddot{w}_2 + \dot{w}_2{}^2) + \frac{0.3}{\ell_1}w_1\ddot{w}_2$$

$$+ \frac{3.0}{\ell_1}(\ddot{w}_1 w_2 + \dot{w}_1\dot{w}_2)\}$$

$$M_2 1.5\frac{\ell_2}{\ell_1}\ddot{w}_1 + M_2\ddot{w}_2 + \lambda_2 w_2 = -M_2\{(\frac{1.2}{\ell_2}w_2 + \frac{1.5}{\ell_1}w_1)\ddot{w}_s + \frac{1.5}{\ell_1}w_1\ddot{w}_1 + \frac{1.2}{\ell_2}w_2\ddot{w}_1$$

$$- \frac{1.2}{\ell_1}(w_1\ddot{w}_2 + \dot{w}_1\dot{w}_2)\}$$

where quadratic nonlinearities only have been retained.

The linearised equations give expressions for the natural frequencies and mode shapes which can be adjusted to give the internal resonance condition exactly, as was the procedure with the three mode model. The equations of motion in normal mode form can then be formulated, and their parameters applied to the more general theory which follows.

Equation (1) on page 10 is the general form of the equations of motion. Denoting the excitation frequency by Ω_f, the cases of immediate interest are

$$\Omega_f = \Omega_1,\ \Omega_f = \Omega_2,\ \Omega_f = \Omega_1 + \Omega_2 \quad \text{where } \Omega_2 = 2\Omega_1$$

A solution of the form $p_r(t) = A_r(t)\cos(\Omega_r t + \phi_r(t)) + f_r\cos\Omega_f t + \varepsilon a_r$ is assumed where $f_r = \frac{F_r}{\Omega_r{}^2 - \Omega_f{}^2}$ is the component of forced response [5]. Application of Struble's technique yields the variational equations. The remainder of this section is a catalogue of the resulting equations for each external frequency condition.

$\underline{\Omega_f = \Omega_1}$

$$-2A_1\Omega_1\dot{\phi}_1 = (\Omega_1^2-\omega_1^2)(A_1+f_1\cos\phi_1) - \frac{\varepsilon}{2}\Big\{P_1A_2\cos(\phi_2-\phi_1) + N_1A_1A_2\cos(\phi_2-2\phi_1)$$

$$+2\Omega_1^2 d_{11} f_1\sin\phi_1 + 2F_1\cos\phi_1\Big\}$$

$$-2\dot{A}_1\Omega_1 = (\Omega_1^2-\omega_1^2)f_1\sin\phi_1 - \frac{\varepsilon}{2}\Big\{-P_1A_2\sin(\phi_2-\phi_1) - N_1A_1A_2\sin(\phi_2-2\phi_1)$$

$$-2\Omega_1^2 d_{11}(A_1+f_1\cos\phi_1) + 2F_1\sin\phi_1\Big\}$$

$$-2A_2\Omega_2\dot{\phi}_2 = (\Omega_2^2-\omega_2^2)A_2 - \frac{\varepsilon}{2}\Big\{P_2A_1\cos(\phi_1-\phi_2) + N_2A_1^2\cos(2\phi_1-\phi_2)$$

$$+M_2\cos\phi_2\Big\}$$

$$-2\dot{A}_2\Omega_2 = -\frac{\varepsilon}{2}\Big\{-P_2A_1\sin(\phi_1-\phi_2) - N_2A_1^2\sin(2\phi_1-\phi_2) + M_2\sin\phi_2$$

$$-2\Omega_2^2 d_{22}A_2\Big\}$$

where :

$$P_1 = k_{12} - f_1\left(\ell_{112}\Omega_2^2 + \ell_{121}\Omega_1^2 - (m_{112}+m_{121})\Omega_1\Omega_2\right) - f_2\left(\ell_{122}(\Omega_1^2+\Omega_2^2) - 2m_{122}\Omega_1\Omega_2\right).$$

$$P_2 = k_{21} - 2f_1\Omega_1^2(\ell_{211}+m_{211}) - f_2\Omega_1^2(\ell_{212}+\ell_{221}+m_{212}+m_{221}).$$

$$N_1 = \Omega_1\Omega_2(m_{112}+m_{121}) - \ell_{112}\Omega_2^2 - \ell_{121}\Omega_1^2.$$

$$N_2 = (-\ell_{211}-m_{211})\Omega_1^2.$$

$$M_2 = f_1^2\Omega_1^2(-\ell_{211}-m_{211}) + f_2^2\Omega_1^2(-\ell_{222}-m_{222}) + f_1f_2\Omega_1^2(-\ell_{212}-\ell_{221}-m_{212}-m_{221})$$

$\underline{\Omega_f = \Omega_2}$

$$-2A_1\Omega_1\dot{\phi}_1 = (\Omega_1^2 - \omega_1^2)A_1 - \frac{\varepsilon}{2}\left\{ P_1 A_1 \cos 2\phi_1 + N_1 A_1 A_2 \cos(\phi_2 - 2\phi_1)\right\}$$

$$-2\dot{A}_1\Omega_1 = -\frac{\varepsilon}{2}\left\{ P_1 A_1 \sin 2\phi_1 - N_1 A_1 A_2 \sin(\phi_2 - 2\phi_1) - 2\Omega_1^2 d_{11} A_1\right\}$$

$$-2A_2\Omega_2\dot{\phi}_2 = (\Omega_2^2 - \omega_2^2)(A_2 + f_2\cos\phi_2) - \frac{\varepsilon}{2}\left\{ N_2 A_1^2 \cos(2\phi_1 - \phi_2) + 2F_2\cos\phi_2\right\}$$

$$-2\dot{A}_2\Omega_2 = (\Omega_2^2 - \omega_2^2) f_2 \sin\phi_2 - \frac{\varepsilon}{2}\left\{ -N_2 A_1^2 \sin(2\phi_1 - \phi_2) + 2F_2 \sin\phi_2 - 2\Omega_2^2 d_{22} A_2\right\}$$

where:

$$P_1 = k_{11} - f_2(\ell_{112}\Omega_2^2 + \ell_{121}\Omega_1^2) - f_1\ell_{111}(\Omega_1^2 + \Omega_2^2) + f_2\Omega_1\Omega_2(m_{112} + m_{121}) + 2f_1\Omega_1\Omega_2 m_{111}$$

$$N_1 = \Omega_1\Omega_2(m_{121} + m_{112}) - (\ell_{121}\Omega_1^2 + \ell_{112}\Omega_2^2)$$

$$N_2 = -(\ell_{211} + m_{211})\Omega_1^2$$

$\underline{\Omega_f = \Omega_1 + \Omega_2}$

$$-2A_1\Omega_1\dot{\phi}_1 = (\Omega_1^2 - \omega_1^2)A_1 - \frac{\varepsilon}{2}\left\{ P_1 A_2 \cos(\phi_1 + \phi_2) + N_1 A_1 A_2 \cos(\phi_2 - 2\phi_1)\right\}$$

$$-2\dot{A}_1\Omega_1 = -\frac{\varepsilon}{2}\left\{ P_1 A_2 \sin(\phi_1 + \phi_2) - N_1 A_1 A_2 \sin(\phi_2 - 2\phi_1) - 2\Omega_1^2 d_{11} A_1\right\}$$

$$-2A_2\Omega_2\dot{\phi}_2 = (\Omega_2^2 - \omega_2^2)A_2 - \frac{\varepsilon}{2}\left\{ P_2 A_1 \cos(\phi_1 + \phi_2) + N_2 A_1^2 \cos(\phi_2 - 2\phi_1)\right\}$$

$$-2\dot{A}_2\Omega_2 = -\frac{\varepsilon}{2}\left\{ P_2 A_1 \sin(\phi_1 + \phi_2) - N_2 A_1^2 \sin(\phi_2 - 2\phi_1) - 2\Omega_2^2 d_{22} A_2\right\}$$

where:

$$P_1 = k_{12} - \Omega_2^2(\ell_{112} f_2 + \ell_{122} f_2) - \Omega_f^2(\ell_{121} f_1 + \ell_{122} f_2) + \Omega_2\Omega_f(f_1(m_{112} + m_{121}) + 2m_{122} f_2)$$

$$P_2 = k_{21} - \Omega_1^2(\ell_{211} f_1 + \ell_{221} f_1) - \Omega_f^2(\ell_{212} f_2 + \ell_{211} f_1) + \Omega_1\Omega_f(f_2(m_{212} + m_{2}) + 2m_{211} f_1)$$

and :

$$N_1 = \Omega_1 \Omega_2 (m_{112} + m_{121}) - (\ell_{112} \Omega_2^2 + \ell_{121} \Omega_1^2)$$

$$N_2 = m_{211} \Omega_1^2 - \ell_{211} \Omega_1^2 .$$

It should be noted that inclusion of the particular integral in the solution leads to terms, of nonlinear origin, accumulating with the terms of parametric origin in the variational equations.

5. FURTHER WORK

The equations listed in the previous section do not lend themselves to analytic solution, and hence recourse is being made to numerical solution using the mathematical model based on the two mode experimental model. Some consideration will be given to the effects of nonlinear elasticity, which are generally taken to be small in comparison with inertial nonlinear effects. The relevance of other nonlinear interaction effects which are little known but which have been the subject of research in this Department will also be considered in relation to the class of problem under investigation.

REFERENCES

[1] Struble, R.A., "Nonlinear Differential Equations" McGraw-Hill, 1962

[2] A.D.S. Barr, and D.J. Nelson. "Autoparametric Interactions in Structures" Symposium on Nonlinear Dynamics, Loughborough University of Tech. 27-28 March 1972

[3] T. Yamamato and A. Saito. "On the vibrations of summed and differential types under parametric excitation" Memoirs of the Faculty of Engineering, Nagoya Univ. 22 May 1970

[4] K.G. Asmis and W.K. Tso "Combination and Internal Resonance in a nonlinear Two degree of freedom system" ASME J.App.Mech. Vol.39, series E, No.3, Sept. 1972.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 18 AFOSR - TR - 76 - 1131 19

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
6 PARAMETRIC AND NONLINEAR MODE INTERACTION BEHAVIOUR IN THE DYNAMICS OF STRUCTURES.

5. TYPE OF REPORT & PERIOD COVERED
9 INTERIM scientific rept.,

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
10 A. D. S. BARR
R. P. ASHWORTH

8. CONTRACT OR GRANT NUMBER(s)
15 VAF AFOSR-~~74~~-2723-74

9. PERFORMING ORGANIZATION NAME AND ADDRESS
UNIVERSITY OF DUNDEE
DEPARTMENT OF MECHANICAL ENGINEERING
SCOTLAND UK DD1 4HN

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
681307
16 AF-9782-04 17 978204
61102F

11. CONTROLLING OFFICE NAME AND ADDRESS
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA
BLDG 410
BOLLING AIR FORCE BASE, D C 20332

12. REPORT DATE
11 1976

13. NUMBER OF PAGES
24 12 26p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
NONLINEAR MODE INTERACTION
DYNAMICS OF STRUCTURES
PARAMETRIC BEHAVIOR
EXPERIMENTAL VIBRATIONS
VARIATIONAL EQUATIONS

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes ~~the continuation of the investigation into~~ the behaviour of structures under internal resonance conditions. Completion of the analysis on the three mode interaction case ~~outlined in the previous report (September 1975)~~ yields the variational equations describing the slowly varying amplitudes and phases of each mode. Computer simulation (C.S.M.P.) of these equations has been carried out. Both parametric and nonlinear terms are found to be of importance in the response of the structure. Further experimental work on the original four mode model, where 'parametric' excitation of two of the modes leads to cont

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED 409 575

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

lpg

eventual participation of all the modes, is presented along with a theoretical approach of a more general nature. Attention is then turned to a less complex two-mode system exhibiting both parametric and nonlinear behaviour.